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CALCULATION OF FLOW AROUND THE LOWER SURFACE OF DELTA WINGS AT WIDE ANGLES OF ATTACK

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CALCULATION OF FLOW AROUND THE LOWER SURFACE OF DELTA WINGS AT WIDE ANGLES OF ATTACK /242*

A. P. Bazzhin

ABSTRACT

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The method of integral relations in the first approximation is used for calculating the flow near the lower surface of delta wings streamlined by a supersonic flow of gas at wide angles of attack. Some calculation results are cited. Author

1. The method of integral relations has already been applied to the investigation of conical flow (ref. 1). The calculations cited in ref. 1 of the flow near circular and elliptical cones at a zero angle of attack, and a circular cone at a small angle of attack ($\alpha = 5^o$), revealed that the method was fairly reliable.

Beginning at some angles of attack, the velocity of the transverse flow on the surface of a circular cone becomes supersonic. A part of the region between

*Numbers given in the margin indicate the pagination in the original foreign text.

the shock wave and the solid body is occupied by the local supersonic zone with the flow on the upper side of the cone no longer affecting the flow near its lower surface.

It may be assumed, on the basis of physical considerations, that in the case of an elliptical cone (with the minor axis of the ellipse in the symmetry plane of the flow), the local supersonic zone on the surface of the solid body will appear earlier, but will be localized in the apex vicinity of the major axis of the ellipse within a wide range of angles of attack.

In the case of an infinite ratio of the elliptical cross section axes, the cone becomes a flat delta. There are three streamlining conditions of such a wing. With the angles of attack ranging from zero to some limit value, the nose compression wave becomes attached to the front edges of the wing. This type of flow near the wing is discussed in detail by numerous authors, particularly in ref. 2 and ref. 3. When the angles of attack exceed the critical angle, the nose compression wave leaves the front edges, but remains attached to the pointed top of the wing. Finally, in the third streamlining condition at still greater angles of attack, the nose compression wave leaves the wing tip, and the conicity of the flow is disrupted.

The second streamlining condition, which is essentially the same in the case of an elliptical cone (including a circular one) and a flat delta wing, is discussed in this study. However, in the latter case the local supersonic zone on the lower wing surface in the lateral plane is constricted to one point representing the front edge.

The entropy on the surface of a solid body plays an important part in the calculation of the flow near conical bodies. In the case of elliptical wings, the following picture of flow lines may be expected to emerge (fig. 1). At zero angle of attack there are two spreading lines on the surface of the elliptical wing; these are the foremost elements of the cone or the front edges of the wing (fig. 1a). All the flow lines converge at point 0 (ref. 4). With the appearance of the angle of attack and its increase, these spreading lines on the surface of the solid body will shift toward the symmetry plane. The speed of that displacement is largely determined by the eccentricity of the elliptical transverse cross section: the smaller the section, the faster the spreading lines will merge into a single line 00' in the symmetry plane on the windward side of /243

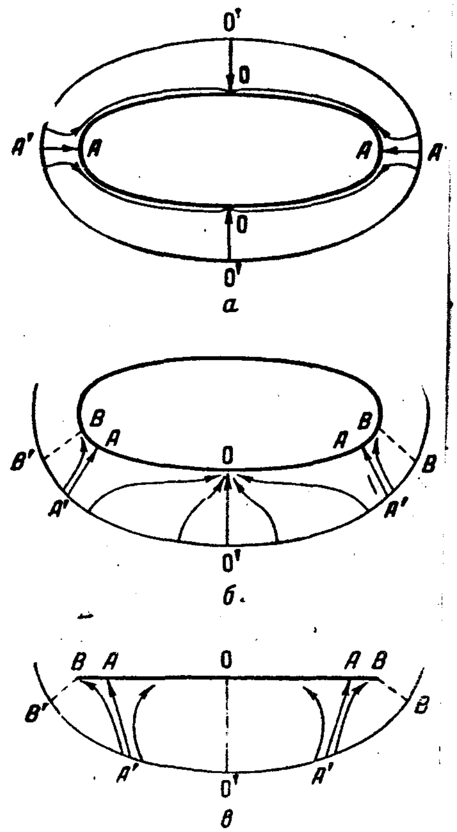

Figure 1

the wing (fig. 1b). A similar flow-line picture will probably take place in the case of a flat delta wing (fig. 1c).

Consequently, the definition of the entropy on a wing surface is an a priori impossibility. The value of the entropy on the surface of a solid body remains a free parameter to be selected in the process of solution, as was indicated in ref. 1.

Taking into account the flow in the transverse section of the wing which is schematically outlined in fig. 1b and 1c, it is possible to raise the problem of defining the flow in the influence zone near the lower surface of the elliptical or flat delta wing.

The solution of this problem by the method of integral relations (in the first approximation) revealed that the number of unknown parameters is equal to the number of the singular points of the approximating system of ordinary differential equations. This has made it possible to find the unique solution to the problem.

2. The problem is solved in a continuous spherical system of coordinates r, ψ, θ (fig. 2). The length of the wing is assumed to be infinite, and the angle of attack a and the velocity of the incident flow (or M_∞ number) are such that the compression wave is attached to the top of the wing. The azimuth angle ψ changes from $\psi = 0$ in the symmetry plane to $\psi = \psi_{max}$, and in the case of a flat delta wing, the ψ_{max} angle corresponds to the front edge. With $\gamma = 1.4$, gas is considered ideal, inviscid, and thermally nonconductive.

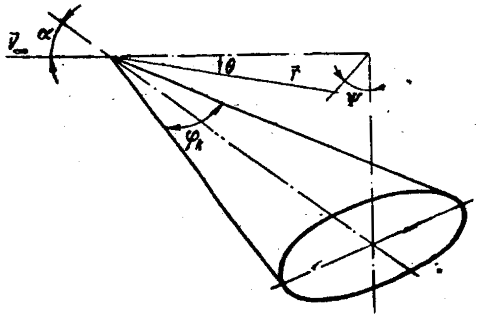

Figure 2

The system of equations describing a conic flow may be recorded as follows:

$$\frac{\partial}{\partial\theta}(p+\rho v^2)\sin\theta+\frac{\partial(\rho vw)}{\partial\psi}=(p+\rho w^2)\cos\theta-3\rho uv\sin\theta,$$
$$\frac{\partial}{\partial\theta}(\rho vw\sin\theta)+\frac{\partial}{\partial\psi}(p+\rho w^2)=-\rho(vw\cos\theta+3uw\sin\theta),$$

$$\frac{\partial}{\partial\theta}(\rho v\sin\theta)+\frac{\partial(\rho w)}{\partial\psi}=-2\rho u\sin\theta,\qquad (1)$$
$$p\rho^{-\gamma}=\varphi(\xi),\quad p=\rho\left(\frac{\gamma+1}{2\gamma}-\frac{\gamma-1}{2\gamma}V^2\right),$$

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where u, v and w are the elements of velocity V modulus in the direction of r, θ, ψ, which are attributed to critical velocity a_{cr}, ρ is the density attributed to the density of the incident flow ρ_∞, p is the pressure attributed to $\rho_\infty a^2_{cr}$, $V^2 = u^2 + v^2 + w^2$, $\varphi(\xi)$ is the entropy function, and the constant along the flow line ξ = const.

The boundary conditions on the compression wave, whose equation is $\theta_1 = \theta_1(\psi)$, can be represented as follows:

$$u_1 = V_\infty\cos\theta_1,$$
$$v_1=\frac{-1+\left(\frac{\gamma-1}{\gamma+1}\right)V_\infty^2(1-\sin^2\theta_1\cos^2\beta)}{V_\infty\sin\theta_1}-V_\infty\sin\theta_1\sin^2\beta,$$

$$w_1 = -\operatorname{tg}\beta\,(v_1 + V_\infty \sin\theta_1),$$
$$\rho_1 = \frac{V_\infty^2 \sin^2\theta_1 \cos^2\beta}{1 - \left(\frac{\gamma-1}{\gamma+1}\right) V_\infty^2 (1 - \sin^2\theta_1 \cos^2\beta)}, \tag{2}$$
$$p_1 = \frac{2}{\gamma+1} V_\infty^2 \sin^2\theta_1 \cos^2\beta - \left(\frac{\gamma-1}{2\gamma}\right)\left(1 - \frac{\gamma-1}{\gamma+1} V_\infty^2\right),$$

where tg $\beta = (1/\sin\theta_1)\ (d\theta_1/d\psi)$. (Index 1 indicates the magnitudes on the compression wave.)

The boundary condition on the wing surface $\theta_0 = \theta_0\ (\psi)$ represents the following nonflow condition:

$$\frac{v_0}{w_0} = \frac{d\theta_0}{\sin\theta_0\, d\psi} = M_0(\psi)$$

(the 0 index indicates the values of the magnitudes on the solid surface). The concrete form of function $M_0\ (\psi)$ depends, of course, on the form of the wing's cross section. For a flat delta wing,

$$M_0(\psi) = \cos\theta_0 \operatorname{tg}\psi.$$

The form of function $M_0\ (\psi)$ for an elliptical wing is considerably more complicated, and is not shown here.

3. The transition to integral relations is made by integrating the first three equations (1) by θ across the shock layer from body $\theta_0\ (\psi)$ to compression wave $\theta_1\ (\psi)$. A linear approximation of integrands produces the following system of ordinary differential equations, which is identical with system (2.5) from ref. 1:

$$\frac{d^2\theta_1}{d\psi^2} = \frac{b_1\sigma_1 + v_0\,[\sigma_2 (b_2 + M_0 b_1) + \overline{A}_2 + Q_2]}{a_1\sigma_1 + v_0\,[\sigma_2 (a_2 + M_0 a_1) - \overline{B}_2]},$$
$$\frac{du_0}{d\psi} = \frac{1}{\rho_0 u_0}\left[(a_2 + M_0 a_1)\frac{d^2\theta_1}{d\psi^2} - (b_2 + M_0 b_1) - \frac{dM_0}{d\psi}\rho_0 v_0 w_0\right],$$

$$\frac{dw_0}{d\psi} = \frac{1}{\left[1 - \frac{\rho_0}{\varkappa p_0}\left(v_0^2 + w_0^2\right)\right]\rho_0} \cdot \left[-\bar{B}_2 \frac{d^2\theta_1}{d\psi^2} + \rho_0\sigma_2\left(u_0 \frac{du_0}{d\psi} + \frac{dM_0}{d\psi} v_0 w_0\right) - Q_2 - \bar{A}_2\right] \tag{3}$$

(the functions included in this system are cumbrously expressed by their own arguments and are not cited here).

The problem can now be solved by integrating system (3) at the value interval of independent variable ψ which begins from the symmetry plane ($\psi = 0$) where /245

$$\frac{d\theta_1}{d\psi} = w_0 = 0.$$

The two unknown parameters of the problem are $\theta_1(0)$ and $u_0(0)$. The values of these parameters are selected by the only method of fulfilling certain conditions at the singular points of the approximating system (3).

One singular point of the approximating system is definite: the point in which the velocity of the transverse flow is equal to the local speed of sound. In the case of an elliptical delta wing, the solution must continuously pass through that point, i.e., the numerator of the last equation of system (3) must also be reduced to zero at that point. The condition required in the case of a flat delta wing is that the velocity of the transverse flow equal the local speed of sound on the front edge of the wing.

The other singular points of the approximating system may be the points in which the denominator of the first equation of system (3) equals zero. It was found that in all the examined alternate versions of the problem, system (3) had only one such singular point. The fulfillment of the continuous solution condition at that singular point, together with the condition at the sonic singular point, made possible a unique definition of both unknown parameters and a unique solution. The selection of the values of the entropy functions on the wing surface was made at the same time.

4. The most characteristic feature of the flow on a delta wing surface is the usual presence of two spreading lines of flow. The results of the calculations of two flat delta wings with full angles at the apex equaling 33° and 34°, with M = 6 and angle of attack 50°, are shown in figs. 3 and 4. Figure 3 shows the position of the compression waves near the lower wing surfaces, and the A'A and $\bar{A}'\bar{A}$ zero lines of flow constructed by the approximate integration of the flow-line equations

$$\frac{d\phi}{d\theta} = \frac{w}{v \sin\theta}.$$

A decrease of the wing angle at the apex results in a shift of the spreading lines toward the symmetry plane, and a simultaneous diminution of the velocity and pressure gradients in their vicinity. The practical constancy of the radial velocity, pressure and density on the median part of the wing is also characteristic of a flat delta wing. /246

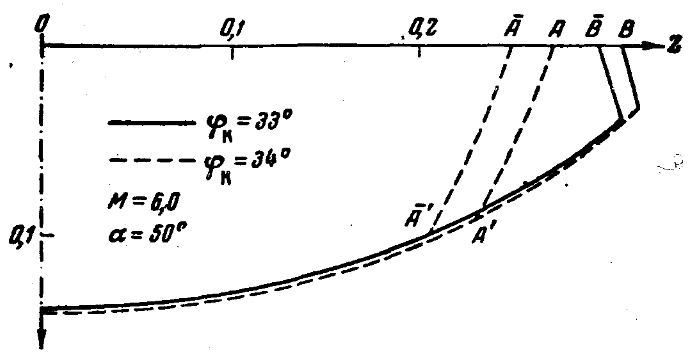

Figure 3

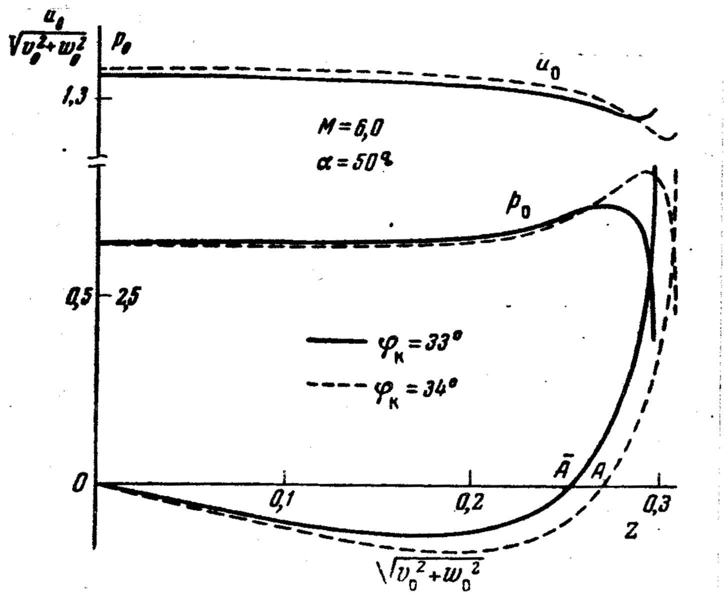

Figure 4

The results obtained may be used for the study of heat transfer on delta wing surfaces at wider angles of attack, now a matter of great practical interest.

Submitted Sept. 2, 1963.

REFERENCES

1. Chushkin, P. I., Shchennikov, V. V. Calculating Certain Conic Flows without Axial Symmetry (Raschet nekotorykh konicheskikh techeniy bez osevoy simmetrii). Inzhenerno-Fizicheskiy Zhurnal, Vol. 3, No. 7, 88-94, 1960.

2. Babayev, D. A. The Numerical Solution of the Problem Involving the Supersonic Flow around the Lower Surface of a Delta Wing (Chislennoye resheniye zadachi obtekaniya nizhney poverkhnosti treugol'nogo kryla sverkhzvukovym potokom). Zhurnal Vychislitel'noy Matematiki i Matematicheskoy Fiziki, Vol. 2, No. 6, 1962.

3. Mel'nikov, D. A. The Supersonic Flow around a Flat Triangular Plate (Obtekaniye sverkhzvukovym potokom ploskoy treugol'noy plastiny). Izvestiya AN SSSR, Dept. of Technological Sciences, Mechanics and Machine Building, XI-XII, 6, 1962.

4. Ferri, A. Supersonic Flow around Circular Cones at Angles of Attack. NACA Rept. 1045, 1951.